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Generational War on Inflation:
Optimal Inflation Rates for the Young and the Old*

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Abstract
How does a grayer society affect the political decision making regarding inflation rates? Is deflation preferred as society ages? In order to answer these questions, we compute the optimal inflation rates for the young and the old respectively and explore how they change with demographic factors, by using a New Keynesian model with overlapping generations. According to our simulation results, there indeed exists a tension between the young and the old on the optimal inflation rates. The optimal inflation rates are different between the young and the old. Also, they can be significantly different from zero, in particular, when heterogeneous impacts from inflation via nominal asset holdings are considered. The optimal inflation rates for the old can be largely negative, reflecting their positive nominal asset holdings as well as lower effective discount factor. Societal aging may exert downward pressure on inflation rates through a politico-economic mechanism.

Keywords: Optimal inflation rates, Societal aging, Heterogeneous agents
JEL classification: E52, J11

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1 Introduction

Japan has been experiencing long-lasting deflation or low inflation rates for more than two decades. Some claim that this is due to the failure of monetary policy with insufficient reaction by the Bank of Japan to declining aggregate demand, based on the idea that "[i]nflation is always and everywhere a monetary phenomenon," by Friedman (1963). On the other hand, because of its long-lasting nature, others point out that chronic deflation or disinflation should have its root in structural issues. An interesting observation in Japan about the relationship between structural factors and nominal developments is that deflation or disinflation started around the mid-1990s when the working age population also started decreasing. Is it a causal relationship or a mere coincidence? In this paper, we offer a new insight on the possible structural relationship between deflation and aging by examining whether the optimal inflation rates are different between the young and the old.

Since aging and low inflation rates are not phenomena intrinsic only to Japan and now observed in many developed economies, several researchers start investigating the possible causal relationship between inflation dynamics and demographic changes. Carvalho and Ferrero (2014) and Fujita and Fujiwara (2016) discuss how societal aging can lead to the declining natural rate of interest, which naturally exerts downward pressure on inflation rates with insufficient monetary policy responses. Carvalho and Ferrero (2014) focus on the demand channel, or *consumption-saving heterogeneity*. Longer longevity induces higher saving rates for self-insurance. Such a saving-for-retirement motive can account for roughly 30% to 50% of the decline in real interest rates in Japan. The decline in fertility rate, however, does not have large impacts. On the other hand, Fujita and Fujiwara (2016) quantify the impact of the supply channel, or *skill (productivity) heterogeneity*. The changes in the demographic structure induce significant low-frequency movements in *per-capita* consumption growth and the real interest rate through the changes in the composition of skilled (old) and unskilled (young) workers. This mechanism can account for roughly 40% of the decline in the real interest rate observed between the 1980s and 2000s in Japan. The key is the declining fertility (labor participation) rate.

Doepke and Schneider (2006) explore the redistribution effect of inflation. Since the old owns more nominal financial assets, they are more vulnerable to unanticipated inflation. On the other hand, surprise inflation can be beneficial to the young because they are borrowers with nominal debt contracts. This conclusion obtained by Doepke and Schneider (2006) also hints that social preference for inflation or deflation may depend on the demographic structure.[1]

We also inquire into whether there is a possible structural relationship between demographic changes and inflation dynamics, but with a different angle. In particular, we compute the optimal inflation rates in the spirit of Schmitt-Grohe and Uribe (2010) with a *politico-economic* consideration following Bassetto (2008), who studies the intergenerational conflicts in tax policy in overlapping generations. Previous studies rather focus on how inflation or a nominal shock affects heterogeneous agents differently. On the other hand, we explore how the optimal inflation rates differ between the young and the old, given the heterogeneous impacts of monetary policy.[2] There are many studies pointing out the heterogeneous impacts of monetary policy, to name a few, Fujiwara and Teranishi (2008), Gornemann et al. (2016), Kaplan et al. (2018), Debortoli and Galí (2017), Wong (2016) and Eichenbaum et al. (2018), but, to the best of our knowledge, ours is the first study to compute the optimal inflation rates for heterogeneous agents.

As comprehensively analyzed in Schmitt-Grohe and Uribe (2010), specifically for Calvo (1983) contract in Ascari (2004) and for Rotemberg (1982) adjustment cost in Bilbiie et al. (2014), inflation rates are not neutral even in the (stochastic) steady states. Inflation rates affect markups in the long-run through such nominal rigidities as considered in Calvo (1983) and Rotemberg (1982) and have impacts on real variables. As a result, optimal inflation rates can be positive or negative depending on the deep parameters. Since the structural parameters differentiate the behavior of the young and the old, the optimal inflation rates are likely to be different between these two agents.[3] In addition, as implied by Doepke and Schneider (2006) and Auclert (2017), the existence of nominal contracts in the financial transactions will lead to asymmetric preference for inflation rates by young and old agents, given the heterogeneity in their nominal asset positions.

We compute the optimal inflation rates for the young and the old, respectively, and also how they change with different demographic settings. For this purpose, we

[1] Auclert (2017) classifies three channels where monetary policy, namely, a surprise nominal shock, causes redistribution: an *earning heterogeneity* channel; a *Fisher channel*; an *interest rate exposure* channel, and finds that all three channels amplify the effects of monetary policy. From a normative perspective, Sheedy (2014) shows that nominal GDP targeting is desirable in an economy with nominal financial contracts, since it can improve risk sharing.

[2] In this regard, our paper's aim is similar to that in Bullard et al. (2012). They construct an overlapping generations model with two assets, capital and money. If old agents have more influence on political decision making, relatively low inflation is chosen because lower inflation reduces the opportunity cost of holding money, and money becomes relatively more attractive and capital accumulation is thus reduced. This raises interest rate, which is preferred by the old since they rely more on capital income than labor income.

[3] This is indeed due to the earnings heterogeneity channel coined by Auclert (2017) in terms of inflation rates.

employ the overlapping generations model with nominal rigidities by Fujiwara and Teranishi (2008), where there are two consumers: the young and the old. They are different in life expectancy and the labor productivity. Unlike the standard overlapping generation model, the transition from the young to the old follows a Markov process with the latter being the absorbing state.[4] Consequently, without resort to highly numerical analysis dealing with the large number of, say, 50 years x 4 quarters = 200 generations, the analysis over the quarterly frequency, where monetary policy is considered to be effective, becomes possible in a tractable framework. In addition, the assumption of RINCE preference *a la* Farmer (1990) enables us to derive the closed form solutions for value functions of agents who are either young or old at any arbitrary time t. We can thus define the optimal inflation rates as those maximizing these values. These altogether enable us to understand the mechanisms behind non-zero optimal inflation rates for heterogeneous agents more intuitively, which is the main aim of this paper.

We first derive the *optimal inflation rates in the long-run*. These are almost equivalent to the *optimal steady state inflation rates*. Any changes in inflation rates, however, alter real variables in the steady state including endogenous state variables in an economy with heterogeneous agents. In such a model, for the proper comparison of welfare by different inflation rate so that we can compute the optimal inflation rates, welfare in the transition given the same initial values for the state variables becomes the metric to be used. Thus, we call the optimal inflation rates in the long-run instead of the optimal steady state inflation rates in this paper.[5]

The optimal inflation rates in the long-run are different both from zero and between the young and the old, implying the importance of demographic factors in determining the target level of inflation. The demographic structure not only determines the level of the optimal inflation rates, but also changes the signs of the optimal inflation rates for the young and the old.

We show that the slope of the long-run Phillips curve, namely whether inflation increases or reduces markups in the steady state, depends on the size of the steady state interest rates, in particular, whether they are higher or lower than the potential growth rate. Changes in the demographic structure naturally lead to the different steady state interest rates. For example, longer life expectancy causes a stronger motive for saving-for-retirement, which lowers steady state interest rates. As a result, the slope of the long-run Phillips curve becomes negative. Then, higher inflation becomes more ben-

[4]The overlapping generation model by Gertler (1999) can be considered as the generalized Blanchard-Rarity model *a la* Blanchard (1985) and Yaari (1965).

[5]The differences in the initial values of endogenous state variables, however, only leads to marginal differences in the optimal inflation rates.

eficial to the old since this reduces marginal costs and therefore increases markups. Opposite effects occur when the interest rates are higher than the potential growth rate in the steady state. To the best of our knowledge, this is the first study to investigate this non-trivial relationship between the optimal inflation rates and demographic factors in the long-run.

The optimal inflation rates in the long-run are, however, only marginally different from zero. When analyzing the optimal inflation rates for the young and the old in the long-run, we deliberately abstract heterogeneous impacts from surprise inflation via nominal asset holdings. If these are considered, the optimal inflation rates for the young and the old can be substantially non-zero from the re-distributional motive through the Fisher channel.

We compute the optimal inflation rates for the young and the old given nominal financial contracts. In the economy with nominal contracts, changes in the target level inflation affect debtors and creditors differently. The central bank needs to set the target level of inflation to take the right balance between short-run gains (or losses) for some particular agents and long-run gains from price stability for all agents. If the former is substantial to some agent, the optimal inflation rate for this agent must be significantly different from zero.

The heterogeneous impacts from surprise inflation via nominal asset holdings turn out to be large. As a result, the optimal inflation rates for old agents, who are net nominal creditors, become largely negative ranging from -0.7% to -5.5% under reasonable parameter calibration. With an increasing number of elderly people, societal aging may exert downward pressure on inflation rates through a politico-economic mechanism.[6]

Naturally, the optimal inflation rates for the young are positive under reasonable parameter calibration, but they are not significantly different from zero, showing a stark contrast to those for old agents, whose optimal inflation rates are significantly negative. Why is there such a huge asymmetry in the optimal inflation rates between the young and the old? To understand the reason behind this asymmetry, we examine which heterogeneity matters for this stark contrast. We first eliminate the heterogeneity in the labor productivity and then in the effective discount factor.

We find that the effective discount factor, namely, life expectancy, is key to this asymmetry. Even without heterogeneity in the labor productivity, this stark contrast somewhat remains. On the other hand, when all agents become almost immortal, the optimal inflation rates for the old become very close to zero, similarly to those for

[6]Katagiri et al. (2019) explains the negative correlation between inflation and aging from a politico-economic perspective. The key mechanism in their paper is the FTPL (Fiscal Theory of the Price Level) and the changes in the tax base via aging. On the other hand, our's is to seek the optimal inflation rates under *monetary dominance*.

young agents. With the presence of the survival rate, old agents become myopic. Benefits from setting non-zero inflation targets stem from the redistribution via nominal contracts. Therefore, they are considered rather short-run gains. On the other hand, costs are price adjustment costs or price dispersion, which persist as long as inflation rates are not zero. Therefore, they are long-run losses. As life expectancy becomes longer (the survival rate becomes higher), old agents become more like young agents and the long-run costs from non-zero inflation rates get larger. Consequently, the optimal inflation rates for old agents become closer to zero even though they lend to the young with nominal fixed contracts.

The remainder of the paper is structured as follows. Section 2 describes the model used in this paper. In Section 3, we show how the optimal inflation rates are different between the young and the old and how they change by different demographic structure. Section 4 incorporates nominal financial contracts and explore their implications on the optimal inflation rates for the young and the old. Section 5 concludes.

2 The Model

In order to investigate the effects of societal aging on the optimal inflation rate, we employ the overlapping generation (OLG) model used in Fujiwara and Teranishi (2008), that extends the analytical framework in Gertler (1999) to incorporate nominal rigidities and monetary policy. Unlike the standard overlapping generation model, the transition from the young to the old follows a Markov process with the latter being the absorbing state. This enables us to understand the mechanisms behind non-zero optimal inflation rates for heterogeneous agents more intuitively in a tractable framework. We also assume perfect foresight throughout all analyses in this paper.

There are six agents in this model economy: two types of consumers: the young and the old; final good producers, intermediate goods producers; a capital producer (financial intermediary); the central bank. The problems which each of the agents except for the central bank faces are as follows.[7] The central bank sets inflation rate in order to maximize welfare.

2.1 Consumers

Each young agent faces a constant probability ω to become old, while each old agent remains in the population with the survival probability γ. Each type of agent is also different in the labor productivity. In the benchmark model, only young agents supply

[7] For details of the derivation, see Gertler (1999) and Fujiwara and Teranishi (2008).

one unit of labor. To be precise, we set old agents' relative labor productivity ξ to be zero. As a result, old agents receive no labor compensation and never work. Appendix A shows the model with endogenous adjustments in intensive margin with non-zero ξ.

Young and old agents are heterogeneous in the effective discount factor and the labor productivity. As a result, the marginal propensities to consume become different between them, which leads to heterogeneous impacts of monetary policy. Also, agents are heterogeneous in asset positions since each agent was born and retired at a different point in time. The heterogeneity in asset positions does not, however, matter for aggregation over young and old agents, respectively. Our model assume RINCE (RIsk Neutrality and Constant Elasticity of Substitution) preference proposed by Farmer (1990). With RINCE preference, consumption function becomes linear in wealth. Thus, only the aggregate wealth matters for aggregate consumption of both young and old agents.

Also, thanks to RINCE preference, we can derive the closed form solutions for value functions of agents who are either young or old at any arbitrary time t. We can thus define the optimal inflation rates as those maximize these values. These altogether enable us to understand the mechanisms behind non-zero optimal inflation rates for heterogeneous agents more intuitively.

There is a perfect annuity market. Therefore, old agents do not face any income uncertainty and enjoy the same *ex post* rate of return as young agents. On the other hand, there is no insurance market for aging risk. In this regard, markets are incomplete in this model.

Let us first discuss the optimization problem of the old, which is assumed to be the absorbing state.

2.1.1 Old

At time t, an old agent, denoted by superscript o, who was born at period j and became old at period k, maximizes welfare:

$$V^o_{j,k,t} := \left\{ \left(C^o_{j,k,t}\right)^\rho + \beta\gamma \left(V^o_{j,k,t+1}\right)^\rho \right\}^{\frac{1}{\rho}},$$

subject to the budget constraint:

$$\frac{A^o_{j,k,t}}{P_t} = \frac{R_{t-1}}{\gamma} \frac{A^o_{j,k,t-1}}{P_t} - C^o_{j,k,t} + D^o_{j,k,t}.$$

C_t, A_t, P_t, and R_t denote consumption, financial assets, aggregate price, and nominal interest, respectively. The old does not supply labor. D_t is the sum of the transfer (or tax) from the government and profits rebated from producers by the ownership of these firms. β and ρ define the common subjective discount factor for both the young and the old, and the inverse of the intertemporal elasticity of substitution, respectively. The next period welfare is discounted by $\beta\gamma$ since the old must take the survival rate into account in maximizing welfare. The rate of return from holding financial assets is divided by γ because of the perfect annuity market among old agents. As a result, bequests are distributed equally among surviving old agents.[8]

2.1.2 Young

A young agent, denoted by the superscript y, who was born at period j maximizes the life time utility:

$$V_{j,t}^{y} := \left\{ \left(C_{j,t}^{y} \right)^{\rho} + \beta \left[\omega V_{j,t+1}^{y} + (1-\omega) V_{j,t+1}^{o} \right]^{\rho} \right\}^{\frac{1}{\rho}},$$

subject to the budget constraint

$$\frac{A_{j,t}^{y}}{P_t} = R_t \frac{A_{j,t-1}^{y}}{P_t} + \frac{W_t}{P_t} - C_{j,t}^{y} + D_{j,t}^{y}.$$

Since each young agent becomes old with probability ω, the next period value is weighted value of the young and the old. In contrast to the old, the young supplies one unit of labor and obtain nominal wage W_t.

2.2 Firms

Final goods, Y_t, are produced by the final goods producers in a competitive market. Differentiated intermediary goods are aggregated by the production technology:

$$Y_t := \left[\int_0^1 \left(Y_{i,t} \right)^{\frac{\kappa-1}{\kappa}} di \right]^{\frac{\kappa}{\kappa-1}}.$$

The parameter κ denotes the elasticity of substitution among differentiated intermediate goods. Given the aggregate price level P_t and the price of each intermediary goods $P_{i,t}$, profit maximization by the final good firm yields the demand for each intermediate goods:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\kappa} Y_t. \tag{1}$$

Firm i in a monopolistically competitive market uses non-differentiated labor $L_{i,t}$

[8] More explicit modeling for the annuity market and the ownership of firms through equity holdings is possible. This will not, however, change our results since perfect foresight is assumed in this paper.

and capital $K_{i,t-1}$ in order to produce differentiated intermediary goods $Y_{i,t}$. The production function of the intermediate goods is given by

$$Y_{i,t} := L_{i,t}^{1-\alpha} K_{i,t-1}^{\alpha}, \tag{2}$$

where α is capital share. Labor is supplied by consumers with nominal wage rate W_t. Capital is rent to intermediary firms at real rate R_t^K from the capital producer. The real cost minimization problem is thus given by

$$\min \left(\frac{W_t}{P_t} L_{i,t} + r_t^K K_{i,t-1} \right)$$

subject to the production function (2). This gives the optimal factor price conditions:

$$\frac{W_t}{P_t} = (1-\alpha)\, \psi_t \, (L_{i,t})^{-\alpha} \, K_{i,t-1}^{\alpha},$$

$$r_t^K = \alpha \psi_t L_{i,t}^{1-\alpha} K_{i,t-1}^{\alpha-1},$$

where ψ_t denotes real marginal costs.

Since each intermediary firm is in a monopolistically competitive market, it chooses price to maximize the profit subject to the Rotemberg (1982) price adjustment cost with the cost parameter ϕ. Instantaneous real profit $\Pi_{i,t}^I$ is given by

$$\Pi_{i,t}^I := (1+\tau)\, \frac{P_{i,t}}{P_t} Y_{i,t} - \psi_t Y_{i,t} - \frac{\phi}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 Y_t.$$

We assume that the intermediaries are owned by consumers, therefore, the profit is rebated equally to all consumers. Let $m_{0,t}$ denote the pricing kernel. Then, the profit maximization problem by price setting becomes

$$\max \sum_{t=0}^{\infty} m_{0,t} \Pi_{i,t}^I,$$

subject to the demand for intermediary goods in equation (1).

Since there are heterogeneous agents, it is not trivial how to define the pricing kernel.[9] In this paper, following Ghironi (2008) and Fujiwara and Teranishi (2008), we only conduct perfect foresight simulations, and therefore all assets yield same rates of return among different agents both *ex ante* and *ex post*. In other words, the profit is discounted by the *risk free rate*. This assumption, however, only matters at the initial period when the target level of inflation is altered.

[9] For the detailed discussion on this issue, see Carceles-Poveda and Coen-Pirani (2009).

In order to eliminate the steady state distortion stemming from monopolistic competition, production subsidy $\tau = \frac{1}{\kappa - 1}$ is assumed. This subsidy is financed by the lump sum tax to both consumers.[10]

2.2.1 Calvo Pricing

We also examine the case with Calvo (1983) pricing. In this case, intermediate goods producer i maximizes

$$\max \sum_{t=0}^{\infty} \lambda^t m_{0,t} \left[(1+\tau) \frac{P_{i,t}}{P_t} Y_{i,t} - \psi_t Y_{i,t} \right],$$

subject again to the downward sloping demand curve in equation (1). λ denotes the Calvo (1983) parameter. Intermediate goods firms can reset the price with unconditional probability of $1 - \lambda$. The model with Calvo (1983) pricing is shown in Appendix B.

2.3 Capital Producer

A capital producer maximizes the profit:

$$\sum_{t=0}^{\infty} m_{0,t} \Pi_t^K,$$

where the instantaneous profit is given by

$$\Pi_t^K := \frac{A_t}{P_t} - R_t \frac{A_{t-1}}{P_t} + r_t^K K_{t-1} - I_t,$$

subject to the capital producing technology:

$$K_t = (1-\delta) K_{t-1} + \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t.$$

A capital producer issues financial assets A_t with nominal rate of return R_t. Such funding from households and the receipts from renting the capital to the intermediate goods producer are allocated to the repayment of borrowing from households and investment I_t. $S(\cdot)$ denotes the investment growth adjustment cost used in Christiano et al. (2005):

$$S(x_t) := s\left(\frac{x_t^2}{2} - x_t + \frac{1}{2} \right).$$

[10] Note that even the lump sum tax is not neutral under heterogeneous consumers.

This capital producer can be also interpreted as a financial intermediary.

2.4 Aggregate Conditions

The financial market clears with

$$q_t K_t = \frac{A_t}{P_t},$$

and

$$A_t = A_t^y + A_t^o,$$

where $A_t^y = \sum_{j=0}^{\infty} A_{j,t}^y$ and $A_t^o = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} A_{j,k,t}^o$. q_t denotes Tobin's Q, which is given by the Lagrange multiplier on the constraint in the capital producer's profit maximization problem.

The good market clears as

$$Y_t = C_t + I_t + \frac{\phi}{2}\left(\pi_t - 1\right)^2 Y_t,$$

where $C_t = C_t^y + C_t^o$, $C_t^y = \sum_{j=0}^{\infty} C_{j,t}^y$ and $C_t^o = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} C_{j,k,t}^o$.

We deliberately assume that the profits are distributed by the relative asset holdings:

$$D_t^o = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} D_{j,k,t}^o = \frac{A_{t-1}^o}{A_{t-1}^y + A_{t-1}^o} D_t, \tag{3}$$

and

$$D_t^y = \sum_{j=0}^{\infty} D_{j,t}^y = \frac{A_{t-1}^y}{A_{t-1}^y + A_{t-1}^o} D_t, \tag{4}$$

where

$$D_t := \Pi_t^I + \Pi_t^K - \tau Y_t = \left[1 - \psi_t - \frac{\phi}{2}\left(\pi - 1\right)^2\right] Y_t + \frac{A_t}{P_t} - R_t \frac{A_{t-1}}{P_t} + r_t^K K_{t-1} - I_t - \tau Y_t,$$

under a symmetric equilibrium. This assumption eliminates re-distributional impacts from inflation stemming from nominal financial contracts. We will first explore the optimal inflation rates in the long-run which is not subject to nominal financial contracts in Section 3 and then incorporate re-distributional channel in Section 4.

The labor market clearing condition is given by

$$L_t := \int_0^{\infty} L_{i,t} \mathrm{d}i = N_t^y,$$

where N_t^y is the population of young workers at time t.

2.5 Equilibrium Conditions

2.5.1 Population

Let N_t^r denote the population of old agents at period t. Then, the population dynamics for the young and the old are, respectively, given by

$$N_{t+1}^y = bN_t^y + \omega N_t^y,$$

and

$$N_{t+1}^o = \gamma N_t^o + (1-\omega) N_t^y,$$

where b denotes the birth rate. The growth rate of (young) population n is given by

$$n := b + \omega - 1.$$

Given these laws of motion, the ratio of the number of old over that of young agents, denoted by Γ_t, evolves as

$$\Gamma_{t+1} := \frac{N_{t+1}^o}{N_{t+1}^y} = \frac{\gamma N_t^o + (1-\omega) N_t^y}{bN_t^y + \omega N_t^y} = \frac{\gamma}{b+\omega}\Gamma_t + \frac{1-\omega}{b+\omega}.$$

At the stationary population, the ratio of the number of old over that of young agents remain constant:

$$\Gamma = \frac{1-\omega}{b+\omega-\gamma}.$$

2.5.2 Equilibrium Conditions in a Monopolistically Competitive Market

From the first order necessary conditions of the above optimization problems, we have the equilibrium conditions under a monopolistically competitive market. All growing variables are de-trended by N_t^y. De-trended variables are denoted by lower case characters. The system of equations except for the monetary policy rule is given by

$$y_t = \left(\frac{k_{t-1}}{1+n}\right)^{\alpha},$$

$$\frac{W_t}{P_t} = (1-\alpha)\, \psi_t \left(\frac{k_{t-1}}{1+n}\right)^{\alpha},$$

$$r_t^K = \alpha\psi_t \left(\frac{k_{t-1}}{1+n}\right)^{\alpha-1},$$

$$d_t = \left[1 - \psi_t - \frac{\phi}{2}(\pi_t - 1)^2\right] y_t + \frac{a_t^y + a_t^o}{P_t} - \frac{R_{t-1}}{(1+n)\pi_t}\frac{a_{t-1}^y + a_{t-1}^o}{P_{t-1}} + \frac{r_t^K}{1+n}k_{t-1} - i_t - \tau y_t, \tag{5}$$

$$(1-\kappa)(1+\tau)y_t + \psi_t \kappa y_t - \phi(\pi_t - 1)\pi_t y_t + \frac{(1+n)\pi_{t+1}}{R_t}\phi(\pi_{t+1} - 1)\pi_{t+1}y_{t+1} = 0, \tag{6}$$

$$k_t = (1-\delta)\frac{k_{t-1}}{1+n} + \left[1 - S\left(\frac{(1+n)i_t}{i_{t-1}}\right)\right] i_t,$$

$$\begin{aligned} 1 \;&=\; q_t \left[1 - S\left(\frac{(1+n)i_t}{i_{t-1}}\right) - S'\left(\frac{(1+n)i_t}{i_{t-1}}\right)\frac{(1+n)i_t}{i_{t-1}}\right] \\ &\quad + \frac{\pi_{t+1}}{R_t} q_{t+1} S'\left(\frac{(1+n)i_{t+1}}{i_t}\right)\left(\frac{(1+n)i_{t+1}}{i_t}\right)^2, \end{aligned}$$

$$q_t = \frac{\pi_{t+1}}{R_t}\left[q_{t+1}(1-\delta) + r_{t+1}^K\right],$$

$$q_t k_t = \frac{a_t^y + a_t^o}{P_t},$$

$$\frac{a_t^o}{P_t} = \frac{R_{t-1}}{(1+n)\pi_t}\frac{a_{t-1}^o}{P_{t-1}} - c_t^o + \frac{a_{t-1}^o}{a_{t-1}^y + a_{t-1}^o}d_t + (1-\omega)\left(\frac{R_{t-1}}{(1+n)\pi_t}\frac{a_{t-1}^y}{P_{t-1}} + \frac{W_t}{P_t} - c_t^y + \frac{a_{t-1}^y}{a_{t-1}^y + a_{t-1}^o}d_t\right),$$

$$c_t^o = \epsilon_t \theta_t \left(\frac{R_{t-1}}{(1+n)\pi_t}\frac{a_{t-1}^o}{P_{t-1}} + \Theta_t^o\right), \tag{7}$$

$$\left(\frac{\epsilon_t \theta_t}{1 - \epsilon_t \theta_t}\right)^{\rho - 1} = \beta \gamma^{1-\rho}\left(\frac{R_t}{\pi_{t+1}}\right)^{\rho}(\epsilon_{t+1}\theta_{t+1})^{\rho-1},$$

$$c_t^y = \theta_t \left(\frac{R_{t-1}}{(1+n)\pi_t}\frac{a_{t-1}^y}{P_{t-1}} + \Theta_t^y\right), \tag{8}$$

$$\left(\frac{\theta_t}{1-\theta_t}\right)^{\rho-1} = \beta\left(\frac{R_t \Phi_{t+1}}{\pi_{t+1}}\right)^{\rho}(\theta_{t+1})^{\rho-1},$$

$$\Theta_t^o = \frac{a_{t-1}^o}{a_{t-1}^y + a_{t-1}^o}d_t + \frac{\gamma \pi_{t+1}}{R_t}\Theta_{t+1}^o, \tag{9}$$

$$\Theta_t^y = \frac{W_t}{P_t} + \frac{a_{t-1}^y}{a_{t-1}^y + a_{t-1}^o}d_t + \frac{\omega \pi_{t+1}}{R_t \Phi_{t+1}}\Theta_{t+1}^y + (1-\omega)\,\epsilon_{t+1}^{\frac{\rho-1}{\rho}}\frac{\pi_{t+1}}{R_t \Phi_{t+1}}\Theta_{t+1}^o,$$

and

$$y_t = c_t^o + c_t^y + \left[1 - S\left(\frac{(1+n)i_t}{i_{t-1}}\right)\right] i_t + \frac{\phi}{2}(\pi_t - 1)^2 y_t, \tag{10}$$

where for simplicity of analysis, we define an auxiliary variable:

$$\Phi_t := \omega + (1-\omega)\,\epsilon_t^{\frac{\rho-1}{\rho}}.$$

π_t denotes gross inflation rates:

$$\pi_t := \frac{P_t}{P_{t-1}}.$$

θ_t and $\epsilon_t\theta_t$ denote the marginal propensity to consume for the young and the old, respectively. Θ_t^y and Θ_t^o denote the aggregated human as well as financial wealth for the young and the old. These equations together with monetary policy, which aims to maximize welfare, determines the equilibrium.

Discussion: Surprise Inflation When the optimal long-run inflation rates are analyzed, our model abstracts the effects of surprise inflation on different consumers through nominal asset holdings analyzed in Doepke and Schneider (2006). The *solved out consumption functions* in equations (7) and (8) are expressed as the product of the marginal propensity to consume and the wealth, which include initial nominal assets divided by the price level at time t. A jump in the price level or inflation seems to affect the wealth of the young and the old differently.

If equations (5) and (9), which determine the profit and the financial wealth for the old respectively, are substituted in equation (7), the solved out consumption function for the old collapses to

$$\begin{aligned} c_t^o \;=\; & \epsilon_t\theta_t\{\frac{a_{t-1}^o}{a_{t-1}^y + a_{t-1}^o}\left\{\left[1-\psi_t-\frac{\phi}{2}\left(\pi_t-1\right)^2\right]y_t + \frac{a_t^y + a_t^o}{P_t} + \frac{r_t^K}{1+n}k_{t-1} - i_t - \tau y_t\right\} \\ & + \frac{\gamma\pi_{t+1}}{(1+n)\,R_t}\Theta_{t+1}^o\}. \end{aligned} \tag{11}$$

The initial real asset position, which is the nominal asset position divided by the price level a_{t-1}^o/P_t, disappears from the wealth. Thus, surprise inflation does not alter the initial real asset position. This irrelevance result stems from our assumption that the profits are shared by the same asset ratio for good producers as well as the capital producer (financial intermediary) in equations (3) and (4), and that all financial assets are identical. We relax this assumption later in Section 4.

2.5.3 Aggregate Value

We can obtain aggregated (de-trended) values for the young and the old at time t as indirect utility as

$$v_t^y = (\theta_t)^{-\frac{1}{\rho}} c_t^y, \tag{12}$$

and

$$v_t^o = (\epsilon_t \theta_t)^{-\frac{1}{\rho}} c_t^o. \tag{13}$$

These are the targets by the central bank to maximize. To be precise, we suppose a situation where there are two political parties: the young party and the old party. The young (old) party represents young (old) consumers at time t and insists that the central banks should commit to a monetary policy to maximize v_t^y (v_t^o).

The assumption of RINCE preference *a la* Farmer (1990) enables us to derive the closed form solutions for value of agents who are either young or old at any arbitrary time t. This greatly simplifies the analysis in this paper and contributes to offering a more intuitive explanation of the non-zero optimal inflation rates.

2.5.4 Monetary Policy

The central bank is equipped with a commitment technology, aiming to maximize welfare defined in equations (12) or (13). Welfare is evaluated at the beginning of transition from the initial state to the one with the new steady state inflation rate:

$$v_0 = f(X_{-1}, \bar{\pi}),$$

where X_t denotes the vector of endogenous state variables and $\bar{\pi}$ is the target level of inflation rate. In a new state, the central bank follows the monetary policy rule:

$$\pi_t = \bar{\pi},$$

and we investigate which $\bar{\pi}$ attains the highest welfare.

Throughout the analyses in this paper, initial states are given by those under zero inflation steady state. As shown in Bilbiie et al. (2014), differences in the initial state variables can lead to incorrect welfare evaluation. The same initial state variables are assumed when comparing welfare.[11]

[11] The optimal inflation rate found in this way depends on the initial state variables. However, even if we set initial state as steady state of ±5% inflation rate, the change is small and our main message still holds.

2.5.5 Calibration

The parameter calibration is shown in Table 1. The model is simulated at a quarterly frequency. The discount factor β and capital depreciation δ are set at $1.04^{-\frac{1}{4}}$ and $1.01^{-\frac{1}{4}} - 1$, respectively. Under our benchmark calibration, we set the parameters for demographic dynamics ω and γ so that on average, each individual works for 45 years and lives as an old agent for 15 years. They are set to $\frac{45\times4-1}{45\times4} = 0.9944$ and $\frac{15\times4-1}{15\times4} = 0.9833$. Population growth rate is set to zero, which implies $b = 1 - \omega = 0.0055$. Other parameters are set to conventional values following Fujiwara and Teranishi (2008). Capital share α and elasticity of substitution of intermediate goods κ are set to $\frac{1}{3}$ and 10, respectively. For the parameter of Rotemberg (1982) cost ϕ, we use 50 so that the New Keynesian Philips Curve of our model matches with the one implied by Calvo (1983) type price setting where one forth of firms change prices in each period on average. Investment adjustment cost s is set 2.48, which is taken from Christiano et al. (2005). Elasticity of intertemporal substitution σ is set to 0.5 which is consistent with Yogo (2004). Also, for the benchmark case, ξ is set to zero.

Table 1: Benchmark Parameter Values

	Parameters	Values
ω	transition probability to old	0.9944
γ	survival rate	0.9833
b	birth rate	$1 - \omega = 0.0055$
β	discount factor	$1.04^{-\frac{1}{4}}$
σ	IES	0.5
ρ	Curvature	$\frac{\sigma-1}{\sigma} = -1$
α	capital share	$\frac{1}{3}$
κ	elasticity of substitution	10
ϕ	Rotemberg cost parameter	50
δ	capital depreciation rate	$1.01^{-\frac{1}{4}} - 1$
s	investment adjustment cost parameter	2.48

3 Optimal Inflation Rate in the Long-Run

In this section, we compute the optimal inflation rates in the long-run for the young and the old without any impact from surprise inflation through nominal financial contracts. We also explore how optimal inflation rates change by different demographic structure by changing such parameters as γ and b.

Let us first explain as to why inflation in the stationary population can have real implications. As discussed in Bilbiie et al. (2014), the New Keynesian Phillips curve in

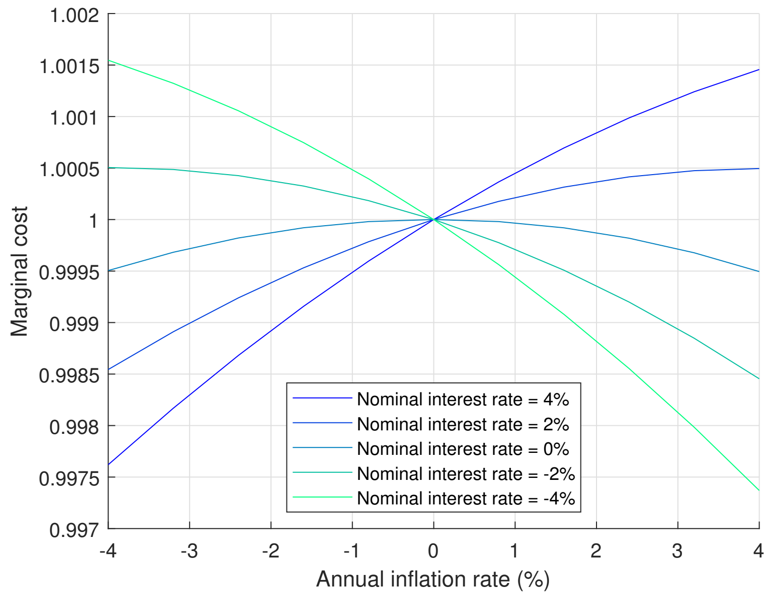


Figure 1: Steady state relationship with Rotemberg

equation (6) implies that fall in inflation rates raises marginal costs when interest rates are low, while rise in inflation rates raises marginal costs when interest rates are high. To highlight this relationship, consider the New Keynesian Phillips curve in the steady state:

$$\psi = \frac{\kappa + \phi\pi\left[-\frac{\pi^2}{R}(1+n) + \frac{\pi}{R}(1+n) + \pi - 1\right]}{\kappa}.$$

Taking the derivative of the right hand side with respect to π gives

$$\phi\frac{\left[-3\frac{\pi^2}{R}(1+n) + 2\frac{\pi}{R}(1+n) + 1\right]}{\kappa}\Bigg|_{\pi=1} = \phi\frac{\left[-\frac{1}{R}(1+n) + 1\right]}{\kappa},$$

which is positive when $R > 1 + n$. Namely, in the steady state, marginal costs rise as inflation increases if and only if nominal interest is larger than the population growth rate. Figure 1 shows the relationship between inflation rates and marginal costs can be upward or downward sloping depending on the level of the steady state real interest rate.

One may cast doubts on the existence of the long-run Phillips curve. Indeed, Benati (2015) shows that there is no clear evidence of a non-vertical trade-off. Benati (2015), however, also points out that uncertainty surrounding the estimates is substantial and therefore, having priors about a reasonable slope in the long-run Phillips curve cannot be falsified.

Intuition behind this long-run Phillips curve with Rotemberg adjustment cost is

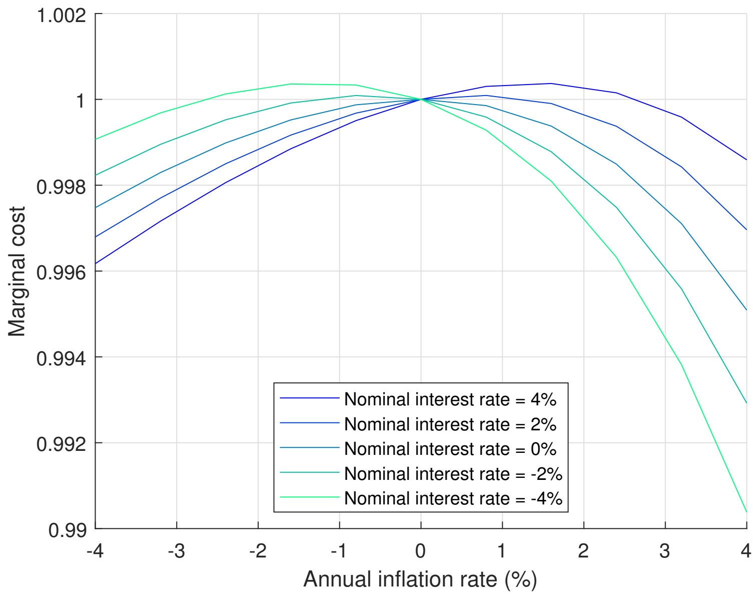


Figure 2: Steady state relationship with Calvo

offered by Lepetit (2017): "Since adjusting prices is costly, firms do not pass on the entirety of movements in marginal costs to prices and current inflation is associated with a reduction in markups. ... [E]xpected future inflation leads firms to set higher markups in order to minimize future price adjustments costs. However, these effects are asymmetric. Since firms discount the future, higher inflation in t has a larger positive impact on marginal cost at time t than a negative impact at time $t-1$. In other words, the model features a positive long-run relationship between inflation and marginal cost." Figure 2 illustrates that similar but slightly different long-run relationship can be observed even with Calvo pricing.

Changes in marginal costs affect the young and the old differently. Figure 3 illustrates how inflation rates and other macroeconomic variables are related in the steady states. Higher marginal costs raise real wage and interest rates. Increase in wage will be welfare-enhancing for young agents since labor compensation is their main source of income. On the other hand, low marginal costs increase welfare for the old because low marginal costs increase firm profits. Since the old has two sources of income, i.e. return from financial assets and profits from the ownership of firms, the strength of this channel depends on the amount of asset holdings. Thus, inflation rates in the stationary population matters for relative welfare between the young and the old through earning heterogeneity.

In order to understand how demographic changes affect optimal inflation rates for both the young and the old, respectively, we examine how changes in (a) life expectancy, (b) the population growth rate, (c) the relative population, and (d) the relative

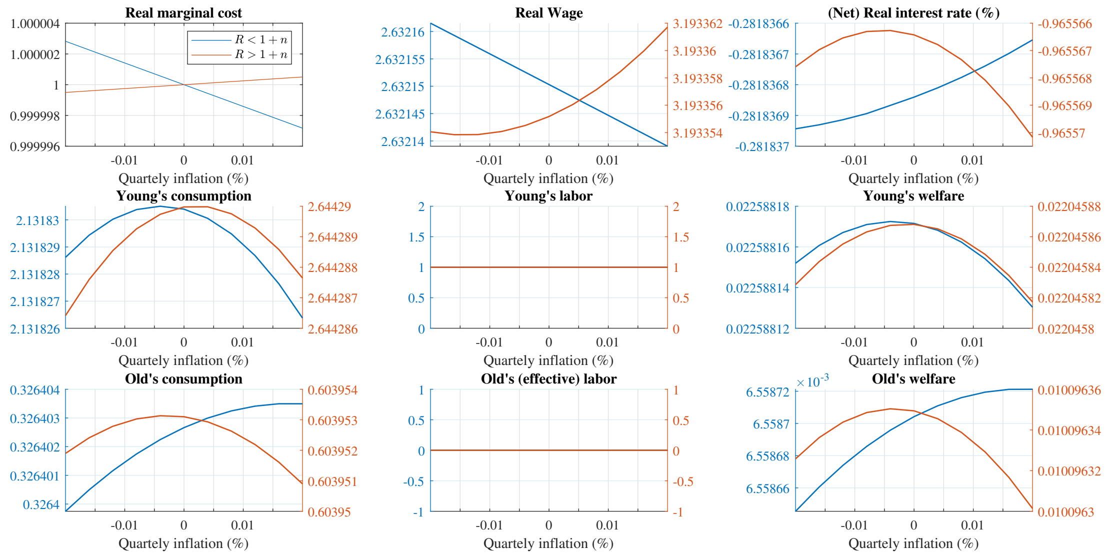


Figure 3: Steady states

asset holdings between the young and the old.

3.1 Life Expectancy

Figure 4 shows how the optimal inflation rates vary depending on the parameter values of γ. The vertical axis shows the optimal annualized inflation rate and the horizontal axis shows life expectancy for the old defined by γ.

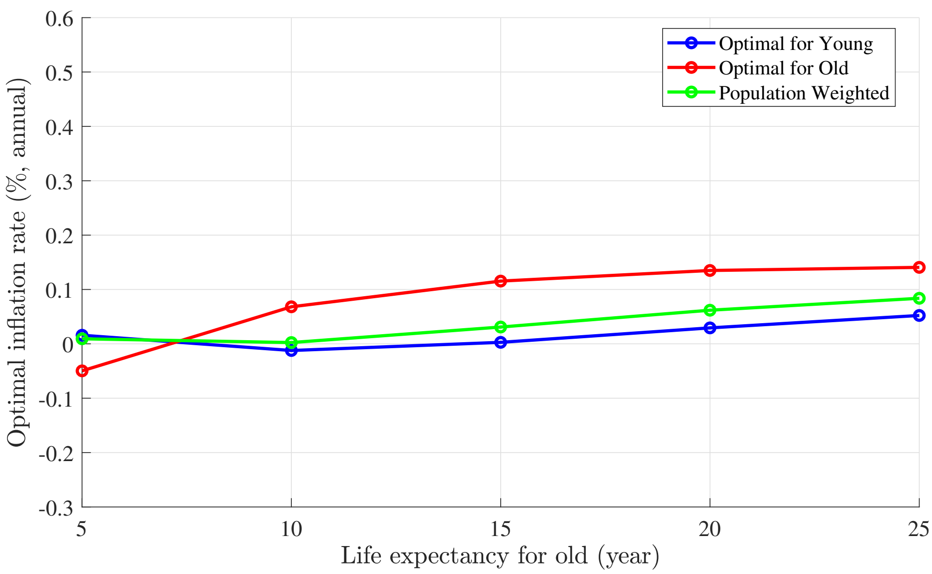


Figure 4: Optimal inflation rate by life expectancy

The young prefers lower inflation rates with longer longevity. This is because interest rates are lowered due to higher motive toward the saving-for-retirement in our economy with overlapping generations. The lower the interest rate (and the higher marginal cost) gets, the higher the wage becomes. Such changes in macroeconomic variables are preferred by the young because they can receive higher earnings, which at the same time increases the marginal propensity to consume under our calibration of intertemporal elasticity of substitution being smaller than unity.

On the other hand, the old wants inflation rates to be higher because profits become larger. Note that if life expectancy conditional on being old is very long, specifically more than 35, the optimal inflation rates for the old decline gradually because their asset holdings become large and returns from financial assets, that are positively correlated with marginal costs, become more important sources of income.

When life expectancy becomes shorter, the young prefers higher inflation rates than the old. In this case, a rise in inflation leads to an increase in marginal costs. This is because interest rates are high due to the relatively small saving-for-retirement motive. The rise in marginal costs increases real wages and real rates of return. Thus, higher inflation rates are preferred by the young while the old can enjoy more consumption from lower inflation rate from higher markups following the exactly opposite logic.

On the other hand, when life expectancy becomes longer than 10 years, young agents' optimal inflation rate starts increasing. This is because their income composition becomes closer to that of the old. The young needs to save more and has stronger incentives to increase welfare when they become old.

3.2 Population Growth

Figure 5 compares the optimal inflation rates by different population growth rate. The horizontal axis is now the annual population growth rate controlled by b. Since there is no technological developments in this economy, the population growth rate corresponds to the potential growth rate.

High (low) population growth rate increases (reduces) interest rates and wage since it increases the capital-labor ratio. We have seen how the long-run inflation rate affects marginal costs in Figure 1. Namely, a positive population growth likely leads to $R < 1 + n$. The young prefers lower inflation to achieve higher wages. The old prefers high inflation to achieve low marginal costs for high profits. On the other hand, when the population growth rate is relatively low, $R > 1 + n$, the signs of the optimal inflation rates flip for each agent.

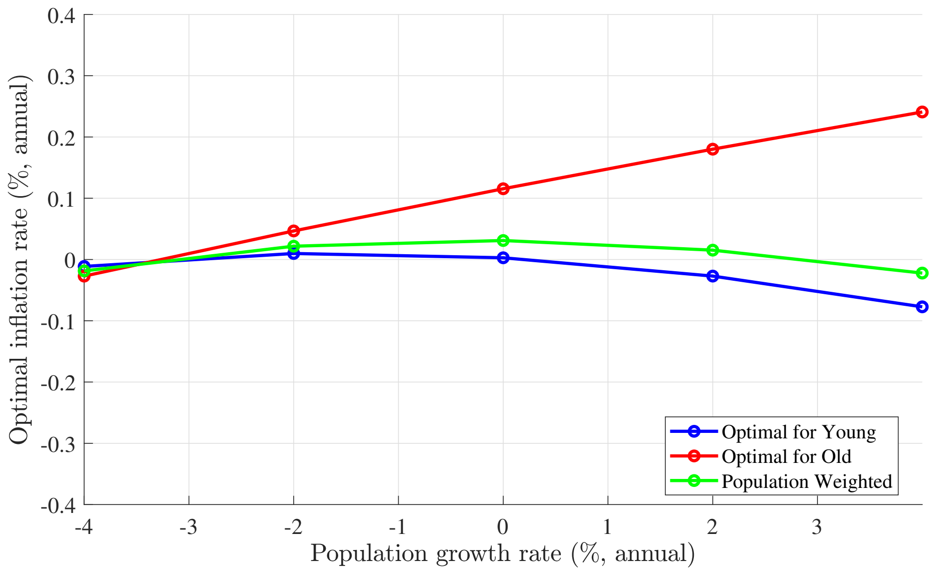


Figure 5: Optimal inflation rate by population growth

3.3 Population Ratio

Figure 6 demonstrates how initial population ratio, N_0^y/N_0^o, affects the optimal inflation rates. Changes in the composition of the population itself do not alter the optimal inflation rates for each young and old agent. The changes in the composition of the population affect only the weighted average of the optimal inflation rates.

3.4 Asset Ratio

Figure 7 illustrates how initial asset distribution affects the optimal inflation rates. In the following figure, we exogenously change the real asset holding ratio at time 0, $A_0^y/(A_0^o + A_0^y)$, from 0.3 to 0.9. As the young holds larger fraction of real assets, interest rates fall because they have a smaller marginal propensity to consume. As illustrated in Figure 1, low rates of return imply that the young prefers lower inflation rates.

Note that our model abstracts the effects of surprise inflation on different consumers through nominal asset positions, which will be investigated in the next section.

3.5 Summary

The optimal inflation rates in the long-run are different both from zero and between the young and the old, implying the importance of the demographic structure in determining the target level of inflation. The demographic structure not only determines the level of the optimal inflation rates, but also changes the signs of the optimal inflation rates for the young and the old. We show that the slope of the long-run Phillips curve,

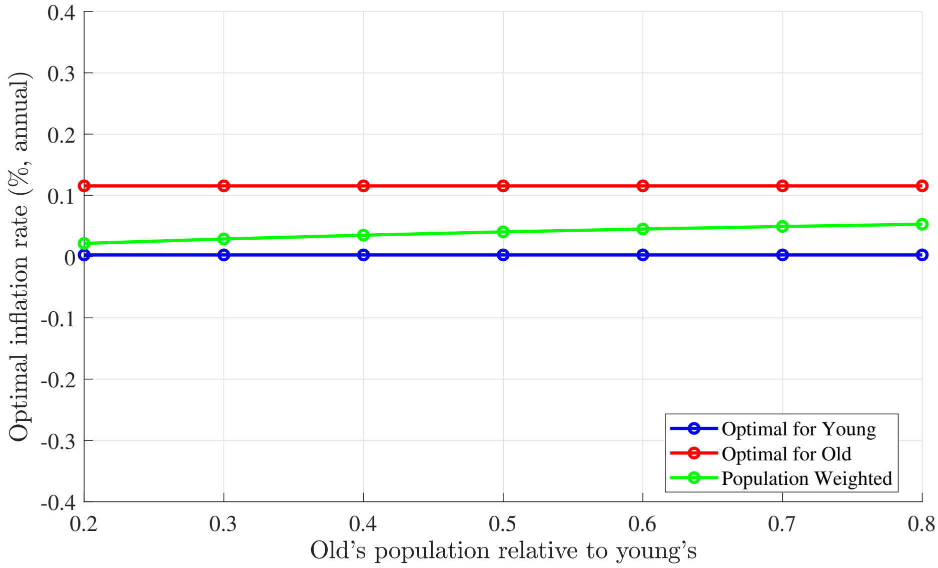


Figure 6: Optimal inflation rate by population ratio

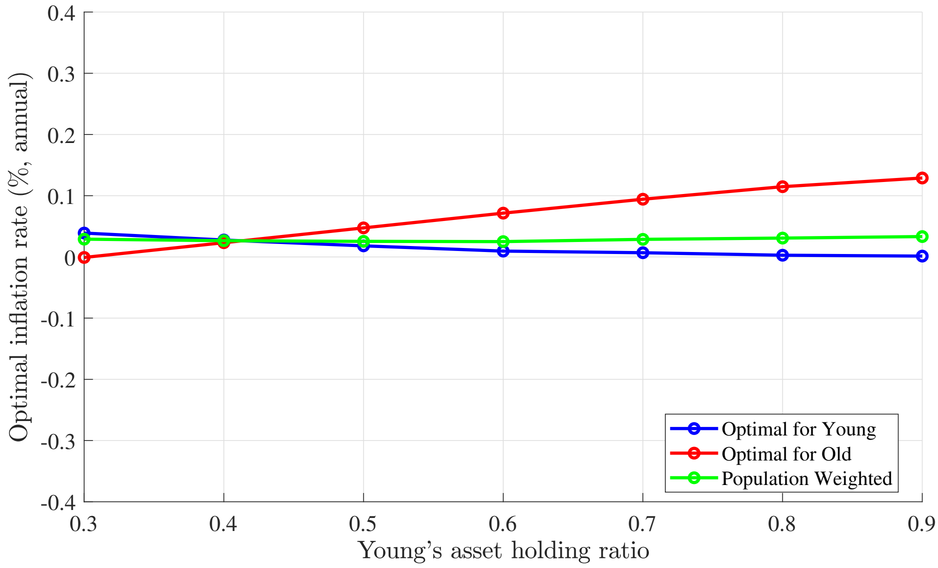


Figure 7: Optimal inflation rate by asset ratio

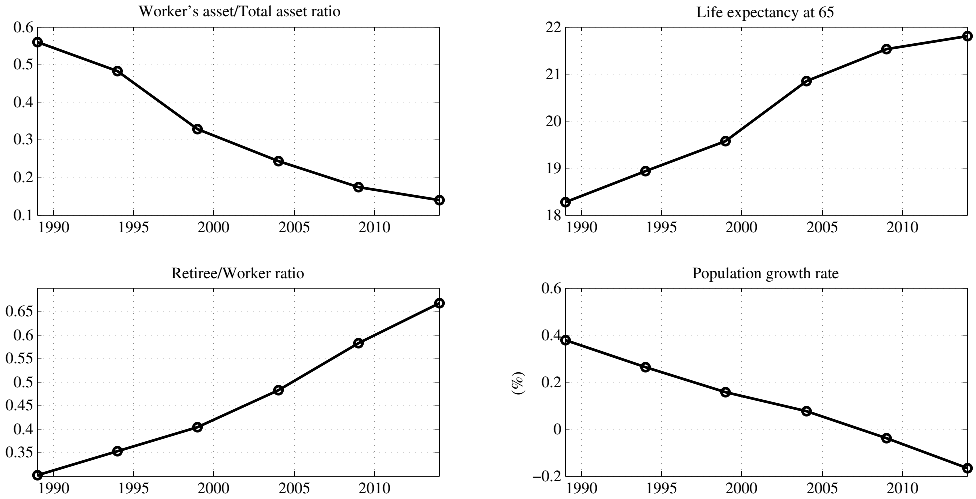


Figure 8: Japanese Data

namely whether inflation increases or reduces markups in the steady state, depends on the size of the steady state interest rates, in particular, whether the steady state real interest rates are higher or lower than the potential growth rate.

Changes in the demographic structure naturally lead to the different steady state interest rates. For example, longer life expectancy leads to a stronger saving-for-retirement motive, which lowers steady state interest rates. As a result, the slope of the long-run Phillips curve becomes negative. Higher inflation becomes more beneficial to the old since this reduces marginal cost and therefore increases markups. The opposite happens when the interest rates are higher than the potential growth rate in the steady state. To the best of our knowledge, this is the first study to investigate this non-trivial relationship between the optimal inflation rates and demographic factors in the long-run.

The optimal long-run inflation rates are, however, only marginally different from zero. Although our model is a stylized model and not calibrated to any specific country, let us check how the demographic variables in the previous subsections have been evolving in Japan. As shown in Figure 8,[12] in Japan, (a) life expectancy becomes longer, (b) the population growth rates becomes slower, (c) as a result, the young/old population ratio has been increasing, and (d) young agent's asset holdings has been decreasing. We cannot, however, find significant fluctuations in such demographic variables as those in the horizontal axes in Figures 4 to 7, namely, even in Japan where the societal aging deepens in an unprecedented manner. This implies that the optimal inflation

[12]The top left panel is the asset holding of the young divided by total asset holding. The top right panel is life expectancy at the age 65. The bottom left panel is the number of old agents divided by number of young agents. The young and the old are respectively defined as the population of age 20 to 65 and the one of age over 65. Population growth rate is plotted in the bottom right panel expressed as the percentile change.

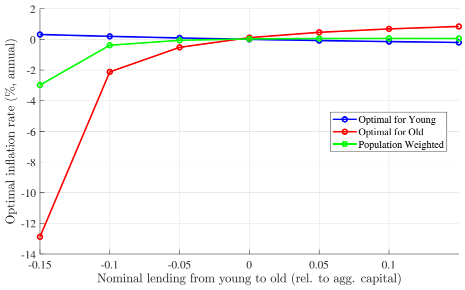
rates cannot be significantly different from zero in the mechanisms considered in this section under reasonable calibration.

Since our focus so far is the long-run optimal inflation rate by the Ramsey planner, we deliberately abstract heterogeneous impacts from surprise inflation via nominal asset holdings. In the model so far, all assets are treated equally and all profits are shared by the young and the old by its nominal asset position. As a result, initial real asset positions, which are nominal asset position divided by the price level, disappear from the wealth in the solved out consumption functions. In reality, the young and the old are different in compositions of nominal assets or liabilities. We explore the implications from nominal contracts on the optimal inflation rates for young and old agents in the next section.

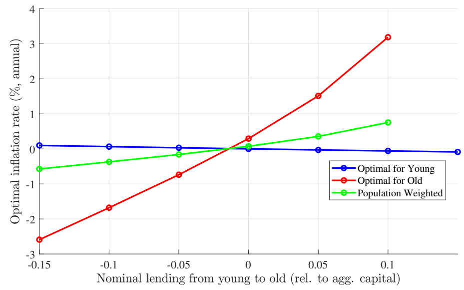
4 Nominal Contracts and Optimal Inflation Rate

Doepke and Schneider (2006) find sizable wealth redistribution among different households from surprise inflation. They show that young and middle-class households with fixed-rate mortgage debt gain the most from surprise inflation. Auclert (2017) coins such a channel the Fisher channel and reports that this channel is important in amplifying the effects of monetary policy. The recent studies by Wong (2016) and Eichenbaum et al. (2018) emphasize the importance of mortgage refinance opportunities. Redistribution through nominal financial contracts have been considered one of the major factors for the heterogeneous impacts of monetary policy. From a normative perspective, Sheedy (2014) shows that nominal GDP targeting is desirable as the stabilization policy in the presence of nominal financial contracts.

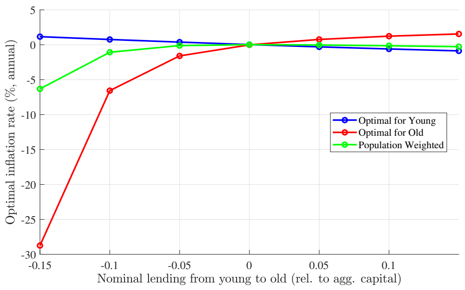
So far, we have abstracted the channel through nominal financial contracts. In this section, we exogenously set the initial nominal asset positions for young and old agents and then investigate the implication for nominal financial contracts on the optimal inflation rate. In particular, we assume nominal lending and borrowing at the initial period between young and old agents. Let B_0^y and B_0^o denote the initial nominal asset positions for young and old agents. Then, the initial real asset position for the young is given by $\left(A_0^y + B_0^y\right)/P_1$ while that for the old is given by $\left(A_0^o + B_0^o\right)/P_1$. Since all profits are shared by the young and the old by their nominal asset positions, distributional effects through the holding of A_0^y and A_0^o are innocuous as explained in Section 2. With nominal lending and borrowing between young and old agents, distributional impacts to nominal shocks emerge. This can be well understood by looking into to the solved out consumption function. The solved out consumption function for the old in



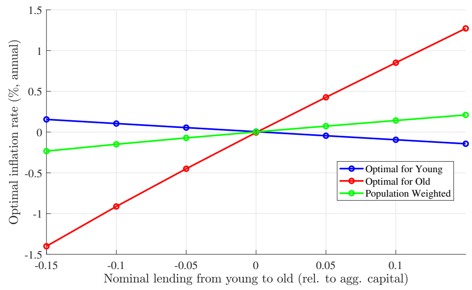
(i) Rotemberg / exo. labor



(ii) Calvo / exo. labor



(iii) Rotemberg / end. labor



(iv) Calvo / end. labor

Figure 9: Nominal contracts and optimal inflation rates

equation (11) now becomes

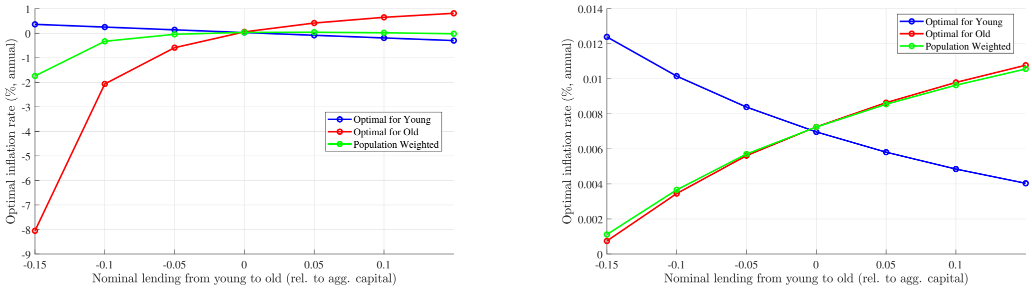
$$
\begin{aligned}
c_t^o &= \epsilon_t \theta_t \{ \frac{b_{t-1}^o}{P_t} + \frac{a_{t-1}^o}{a_{t-1}^y + a_{t-1}^o} \left\{ \left[1 - \psi_t - \frac{\phi}{2} (\pi_t - 1)^2 \right] y_t + \frac{a_t^y + a_t^o}{P_t} + \frac{r_t^K}{1+n} k_{t-1} - i_t - \tau y_t \right\} \\
&+ \frac{\gamma \pi_{t+1}}{(1+n) R_t} \Theta_{t+1}^o \}.
\end{aligned}
$$

Thus, surprise change in P_t affects young and old agents differently through the first term in the bracket b_{t-1}^o / P_t. The market clearing condition for this financial asset is given by

$$B_0^y + B_0^o = 0.$$

We will first see how optimal inflation rates for the young and the old change as the ratio B_0^y / K_0 or B_0^o / K_0 is altered. The optimal inflation rates will not likely be zero. The central bank needs to take the right balance between the long-run price stability (zero inflation rates) and the redistribution needs from each agent. Second, we discuss how the optimal inflation rate is different from zero with the empirically plausible level of nominal financial contracts found in the data.

Figure 9 illustrates how optimal inflation rates change with initial nominal asset positions B_0^y / K_0 for (i) Rotemberg adjustment cost with exogenous labor, (ii) Calvo pricing with exogenous labor, (iii) Rotemberg adjustment cost with endogenous labor, and (iv) Calvo pricing with endogenous labor. The more young agents borrow from the old, the lower the optimal inflation rates for old agents and the higher the optimal



No productivity difference | Immortal

Figure 10: Nominal contracts and optimal inflation rate: less heterogeneity

inflation rates for young agents. We observe similar tendencies when old agents borrow more from the young. There, the optimal inflation rates for the young becomes lower while that for the old becomes higher. These are consistent with a motive to reduce the amount of debt through surprise inflation. There, however, exists significant asymmetry. The optimal inflation rate for the old, when young agents are borrowing and old agents are lending, is hugely negative but the optimal inflation for the young is only slightly positive.

To understand the reason behind this asymmetry, we conduct two additional experiments. The optimal inflation rates are computed when there is no difference in labor productivity or agents are almost immortal, respectively. Figure 10 shows results from these two experiments. We only examine the case with Rotemberg adjustment cost and exogenous labor supply. Asymmetry still remains even without heterogeneity in the labor productivity, but it almost disappears when all agents become almost immortal.

How the non-zero inflation rates are preferred by the different agents depends crucially on life expectancy. With the presence of the survival rate, old agents become myopic. Benefits from setting non-zero inflation targets come from the redistribution via nominal contracts and therefore they are rather short-run gains. On the other hand, costs are Rotemberg price adjustment costs or price dispersion, which persist as long as inflation rates are non-zero. Therefore, they are long-run losses. As life expectancy becomes longer, old agents become more like young agents and the long-run costs from non-zero inflation rates become larger. Consequently, the optimal inflation rates for old agents become closer to zero even though they lend to the young.

Then, the natural question is how large the optimal inflation rates should be for young and old agents respectively given the realistic level of nominal financial asset positions observed in the data? It is not a trivial task to define which are nominal or real assets, which contracts are fixed nominal or flexible, and, in addition, who incurs the costs from surprise inflation given complicated ownership of firms and delegation of financial asset managements. Thus, we borrow the estimate by Doepke and Schneider (2006), in particular, those in "Table 1 Net Nominal Position of U.S. Households in 1989." Since our model only has two agents, the numbers in Table 1 in Doepke and Schneider (2006) corresponds to -0.08 (-8%) as the empirically plausible value of B_0^y/K_0. Table 2 compares the optimal inflation rates for young and old agents in four cases examined above: (i) Rotemberg adjustment cost with exogenous labor; (ii) Calvo pricing with exogenous labor; (iii) Rotemberg adjustment cost with endogenous labor; (iv) Calvo pricing with endogenous labor.

Table 2: Optimal Inflation Rates

		young	old
Exogenous labor	Rotemberg	0.1%	-1.9%
	Calvo	0.1%	-1.4%
Endogenous labor	Rotemberg	0.5%	-5.5%
	Calvo	0.1%	-0.7%

The optimal inflation rates for the old is significantly different from zero and very negative.

Welfare gains from optimal inflation rates are sizable. Under the optimal inflation rate for the young, young consumption is higher by 0.08% with Rotemberg adjustment cost and 0.05% with Calvo pricing than under the optimal inflation rate for the old. Under the optimal inflation rate for the old, old consumption is higher by 0.08% with the Rotemberg adjustment cost and 0.16% with Calvo pricing than under the optimal inflation rate for the young. The tension between the young and the old can be very tight on the optimal choice of the target level of inflation rates. Aging has significant politico-economic implications to the optimal conduct of monetary policy.

5 Conclusion

The optimal inflation rates in the long-run are different both from zero and between the young and the old. The demographic structure can potentially have significant implications for optimal inflation rates for the young and the old. It not only determines the level of the optimal inflation rates, but also changes the sign of the optimal inflation rates for the young and the old, leading to the non-trivial relationship between the optimal inflation rates and demographic factors in the long-run. Also, we find that the optimal inflation rates are significantly different from zero, in particular, when heterogeneous impacts from surprise inflation via nominal asset holdings are considered. The optimal inflation rates for the old given their positive nominal asset holdings can be largely negative. This largely negative optimal inflation rates for the old is caused via higher discount factor reflecting shorter life expectancy of old agents.

We deliberately use a tractable framework to investigate the optimal inflation rates for the young and the old. This is because the main aim of this paper is to understand the mechanisms behind non-zero optimal inflation rates for heterogeneous agents more intuitively in a tractable framework. The simplification, however, of course comes with costs. Rather strong assumptions needed to be imposed, such as RINCE preference. More quantitatively demanding exercises using the overlapping generations model with less restrictions will offer much sharper policy prescriptions on the optimal inflation rates for heterogeneous agents: Many agents and more financial assets are considered; the parameter calibration is based on more detailed data analysis on the household's balance sheets, in particular, how components of assets and liabilities are constrained by nominal contracts, or which assets are owned by each agent directly. This agenda is left for future studies.

References

ASCARI, GUIDO (2004): "Staggered Prices and Trend Inflation: Some Nuisances," *Review of Economic Dynamics*, 7, 642–667.

AUCLERT, ADRIEN (2017): "Monetary Policy and the Redistribution Channel," NBER Working Papers 23451, National Bureau of Economic Research, Inc.

BASSETTO, MARCO (2008): "Political Economy of Taxation in an Overlapping-Generations Economy," *Review of Economic Dynamics*, 11, 18–43.

BENATI, LUCA (2015): "The long-run Phillips curve: A structural VAR investigation," *Journal of Monetary Economics*, 76, 15–28.

BILBIIE, FLORIN O., IPPEI FUJIWARA, AND FABIO GHIRONI (2014): "Optimal monetary policy with endogenous entry and product variety," *Journal of Monetary Economics*, 64, 1–20.

BLANCHARD, OLIVIER J (1985): "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93, 223–47.

BULLARD, JAMES, CARLOS GARRIGA, AND CHRISTOPHER J. WALLER (2012): "Demographics, redistribution, and optimal inflation," *Federal Reserve Bank of St. Louis Review*, 94, 419–440.

CALVO, GUILLERMO A (1983): "Staggered prices in a utility-maximizing framework," *Journal of monetary Economics*, 12, 383–398.

CARCELES-POVEDA, EVA AND DANIELE COEN-PIRANI (2009): "Shareholders' Unanimity With Incomplete Markets," *International Economic Review*, 50, 577–606.

CARVALHO, CARLOS AND ANDREA FERRERO (2014): "What Explains Japan's Persistent Deflation?" *Manuscript, University of Oxford, August*.

CHRISTIANO, LAWRENCE J, MARTIN EICHENBAUM, AND CHARLES L EVANS (2005): "Nominal rigidities and the dynamic effects of a shock to monetary policy," *Journal of political Economy*, 113, 1–45.

DEBORTOLI, DAVIDE AND JORDI GALÍ (2017): "Monetary policy with heterogeneous agents: Insights from TANK models," *Manuscript, September*.

DOEPKE, MATTHIAS AND MARTIN SCHNEIDER (2006): "Inflation and the redistribution of nominal wealth," *Journal of Political Economy*, 114, 1069–1097.

EICHENBAUM, MARTIN, SERGIO REBELO, AND ARLENE WONG (2018): “State Dependent Effects of Monetary Policy: the Refinancing Channel,” NBER Working Papers 25152, National Bureau of Economic Research, Inc.

FARMER, ROGER E. A. (1990): “RINCE Preferences,” *The Quarterly Journal of Economics*, 105, 43–60.

FRIEDMAN, MILTON (1963): *Inflation: Causes and consequences*, Asia Publishing House.

FUJITA, SHIGERU AND IPPEI FUJIWARA (2016): “DECLINING TRENDS IN THE REAL INTEREST RATE AND INFLATION: THE ROLE OF AGING,” Tech. rep., Federal Reserve Bank of Philadelphia.

FUJIWARA, IPPEI AND YUKI TERANISHI (2008): “A dynamic new Keynesian life-cycle model: Societal aging, demographics, and monetary policy,” *Journal of Economic Dynamics and Control*, 32, 2398–2427.

GERTLER, MARK (1999): “Government debt and social security in a life-cycle economy,” *Carnegie-Rochester Conference Series on Public Policy*, 50, 61–110.

GHIRONI, FABIO (2008): “The role of net foreign assets in a New Keynesian small open economy model,” *Journal of Economic Dynamics and Control*, 32, 1780–1811.

GORNEMANN, NILS, KEITH KUESTER, AND MAKOTO NAKAJIMA (2016): “Doves for the Rich, Hawks for the Poor? Distributional Consequences of Monetary Policy,” International Finance Discussion Papers 1167, Board of Governors of the Federal Reserve System (U.S.).

KAPLAN, GREG, BENJAMIN MOLL, AND GIOVANNI L. VIOLANTE (2018): “Monetary Policy According to HANK,” *American Economic Review*, 108, 697–743.

KATAGIRI, MITSURU, HIDEKI KONISHI, AND KOZO UEDA (2019): “Aging and Deflation from a Fiscal Perspective,” *Journal of Monetary Economics*.

LEPETIT, ANTOINE (2017): “The Optimal Inflation Rate with Discount Factor Heterogeneity,” Working Papers hal-01527816, HAL.

ROTEMBERG, JULIO J (1982): “Sticky Prices in the United States,” *Journal of Political Economy*, 90, 1187–1211.

SCHMITT-GROHE, STEPHANIE AND MART?N URIBE (2010): “The Optimal Rate of Inflation,” in *Handbook of Monetary Economics*, ed. by B. M. Friedman and M. Woodford, Elsevier, vol. 3 of *Handbook of Monetary Economics*, chap. 13, 653–722.

SHEEDY, KEVIN D. (2014): "Debt and Incomplete Financial Markets: A Case for Nominal GDP Targeting," *Brookings Papers on Economic Activity*, 45, 301–373.

WONG, ARLENE (2016): "Population aging and the transmission of monetary policy to consumption," 2016 Meeting Papers 716, Society for Economic Dynamics.

YAARI, MENAHEM E (1965): "Uncertain lifetime, life insurance, and the theory of the consumer," *The Review of Economic Studies*, 32, 137–150.

YOGO, MOTOHIRO (2004): "Estimating the Elasticity of Intertemporal Substitution When Instruments Are Weak," *The Review of Economics and Statistics*, 86, 797–810.

Appendix

A Variable Labor Supply

When labor supply decision is endogenous, the old maximizes welfare:

$$V^o_{j,k,t} := \left\{ \left[\left(C^o_{j,k,t}\right)^{\nu} \left(1 - L^o_{j,k,t}\right)^{1-\nu} \right]^{\rho} + \beta\gamma \left(V^o_{j,k,t+1}\right)^{\rho} \right\}^{\frac{1}{\rho}},$$

subject to the budget constraint:

$$\frac{A^o_{j,k,t}}{P_t} = \frac{R_{t-1}}{\gamma} \frac{A^o_{j,k,t-1}}{P_t} - C^o_{j,k,t} + \frac{W_t}{P_t} \xi L^o_{j,k,t} + D^o_{j,k,t},$$

while the young maximizes welfare:

$$V^y_{j,t} := \left\{ \left[\left(C^y_{j,t}\right)^{\nu} \left(1 - L^y_{j,t}\right)^{1-\nu} \right]^{\rho} + \beta \left[\omega V^y_{j,t+1} + (1-\omega) V^o_{j,t+1} \right]^{\rho} \right\}^{\frac{1}{\rho}},$$

subject to the budget constraint

$$\frac{A^y_{j,t}}{P_t} = R_t \frac{A^y_{j,t-1}}{P_t} + \frac{W_t}{P_t} L_{j,t} - C^y_{j,t} + D^y_{j,t}.$$

As a result of the endogenous labor supply, the labor market clears as

$$L_t = L^y_t + \xi L^o_t.$$

When the labor is endogenous, the system of equations except for monetary policy rule is now given by

$$y_t = \left(\frac{k_{t-1}}{1+n}\right)^{\alpha},$$

$$\frac{W_t}{P_t} = (1-\alpha)\,\psi_t l_t^{-\alpha} \left(\frac{k_{t-1}}{1+n}\right)^{\alpha},$$

$$r^K_t = \alpha\psi_t \left(\frac{k_{t-1}}{1+n}\right)^{\alpha-1},$$

$$d_t = \left[1 - \psi_t - \frac{\phi}{2}(\pi_t - 1)^2\right] y_t + \frac{a^y_t + a^o_t}{P_t} - \frac{R_{t-1}}{(1+n)\,\pi_t} \frac{a^y_{t-1} + a^o_{t-1}}{P_{t-1}} + r^K_t \frac{k_{t-1}}{1+n} - i_t - \tau y_t,$$

$$(1-\kappa)(1+\tau)\, y_t + \psi_t \kappa y_t - \phi(\pi_t - 1)\,\pi_t y_t + \frac{(1+n)\,\pi_{t+1}}{R_t} \phi(\pi_{t+1} - 1)\,\pi_{t+1} y_{t+1} = 0,$$

$$k_t = (1-\delta)\frac{k_{t-1}}{1+n} + \left[1 - S\left(\frac{(1+n)\,i_t}{i_{t-1}}\right)\right] i_t,$$

$$\begin{aligned} 1 &= q_t\left[1 - S\left(\frac{(1+n)\,i_t}{i_{t-1}}\right) - S'\left(\frac{(1+n)\,i_t}{i_{t-1}}\right)\frac{(1+n)\,i_t}{i_{t-1}}\right] \\ &\quad + \frac{\pi_{t+1}}{R_t} q_{t+1} S'\left(\frac{(1+n)\,i_{t+1}}{i_t}\right)\left(\frac{(1+n)\,i_{t+1}}{i_t}\right)^2, \end{aligned}$$

$$q_t = \frac{\pi_{t+1}}{R_t}\left[q_{t+1}(1-\delta) + r^K_{t+1}\right],$$

$$q_t k_t = \frac{a^y_t + a^o_t}{P_t},$$

$$\begin{aligned} \frac{a^o_t}{P_t} &= \frac{R_{t-1}}{(1+n)\,\pi_t}\frac{a^o_{t-1}}{P_{t-1}} + \frac{W_t}{P_t}\xi l^o_t - c^o_t + \frac{a^o_{t-1}}{a^y_{t-1} + a^o_{t-1}} d_t \\ &\quad + (1-\omega)\left(\frac{R_{t-1}}{(1+n)\,\pi_t}\frac{a^y_{t-1}}{P_{t-1}} + \frac{W_t}{P_t} l^y_t - c^y_t + \frac{a^y_{t-1}}{a^y_{t-1} + a^o_{t-1}} d_t\right), \end{aligned}$$

$$c^o_t = \epsilon_t\theta_t\left(\frac{R_{t-1}}{(1+n)\,\pi_t}\frac{a^o_{t-1}}{P_{t-1}} + \Theta^o_t\right),$$

$$\left(\frac{\epsilon_t\theta_t}{1-\epsilon_t\theta_t}\right)^{\rho-1} = \beta\gamma^{1-\rho}\left(\frac{R_t}{\pi_{t+1}}\right)^{\rho\nu}\left(\frac{\pi_{t+1}W_{t+1}P_t}{P_{t+1}W_t}\right)^{\rho(\nu-1)}\left(\epsilon_{t+1}\theta_{t+1}\right)^{\rho-1},$$

$$c^y_t = \theta_t\left(\frac{R_{t-1}}{(1+n)\,\pi_t}\frac{a^y_{t-1}}{P_{t-1}} + \Theta^y_t\right),$$

$$\left(\frac{\theta_t}{1-\theta_t}\right)^{\rho-1} = \beta\left(\frac{R_t\Phi_{t+1}}{\pi_{t+1}}\right)^{\rho\nu}\left(\frac{\pi_{t+1}W_{t+1}P_t}{P_{t+1}W_t}\right)^{\rho(\nu-1)}\left(\theta_{t+1}\right)^{\rho-1},$$

$$\Theta^o_t = \frac{W_t}{P_t}\xi l^o_t + \frac{a^o_{t-1}}{a^y_{t-1} + a^o_{t-1}} d_t + \frac{\gamma\pi_{t+1}}{R_t}\Theta^o_{t+1},$$

$$\Theta^y_t = \frac{W_t}{P_t} l^y_t + \frac{a^y_{t-1}}{a^y_{t-1} + a^o_{t-1}} d_t + \frac{\omega\pi_{t+1}}{R_t\Phi_{t+1}}\Theta^y_{t+1} + (1-\omega)\,\epsilon_{t+1}^{\frac{\rho-1}{\rho}}\xi^{\nu-1}\frac{\pi_{t+1}}{R_t\Phi_{t+1}}\Theta^o_{t+1},$$

$$y_t = c^o_t + c^y_t + \left[1 - S\left(\frac{(1+n)\,i_t}{i_{t-1}}\right)\right] i_t + \frac{\phi}{2}\left(\pi_t - 1\right)^2 y_t,$$

$$\Phi_t := \omega + (1-\omega)\,\epsilon_t^{\frac{\rho-1}{\rho}}\xi^{\nu-1},$$

$$l_t^o = \Gamma_t - \frac{1-\nu}{\nu} c_t^o \frac{P_t}{\xi W_t},$$

$$l_t^y = 1 - \frac{1-\nu}{\nu} c_t^y \frac{P_t}{W_t},$$

and

$$l_t = l_t^y + \xi l_t^o.$$

Values to be targeted by the central bank are altered as

$$v_t^y = (\theta_t)^{-\frac{1}{\rho}} c_t^y \left(\frac{1-\nu}{\nu} \frac{P_t}{W_t} \right)^{1-\nu},$$

and

$$v_t^o = (\epsilon_t \theta_t)^{-\frac{1}{\rho}} c_t^o \left(\frac{1-\nu}{\nu} \frac{P_t}{\xi W_t} \right)^{1-\nu}.$$

B Calvo Pricing

When Calvo (1983) pricing is employed instead of Rotemberg (1982) adjustment cost, equation (6), which depicts the new Keynesian Phillips curve, is replaced by three equations below:

$$\left(\frac{1-\lambda \pi_t^{\kappa-1}}{1-\lambda} \right)^{\frac{1}{1-\kappa}} F_t = G_t,$$

$$F_t = 1 + \lambda \frac{\pi_{t+1}}{R_t} \frac{Y_{t+1}}{Y_t} \pi_{t+1}^{\kappa-1} F_{t+1},$$

and

$$G_t = \frac{\kappa}{(1-\tau)(\kappa-1)} + \lambda \frac{\pi_{t+1}}{R_t} \frac{Y_{t+1}}{Y_t} \pi_{t+1}^{\kappa} G_{t+1}.$$

Also, the resource constraint in equation (10) is replaced by

$$y_t = \Delta_t \left[c_t^o + c_t^y + i_t + \frac{\phi}{2} (\pi_t - 1)^2 y_t \right],$$

where Δ_t denotes the price dispersion term:

$$\Delta_t := \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\kappa} \mathrm{d}i = \lambda \pi_t^{\kappa} + (1-\lambda) \left(\frac{1-\lambda \pi_t^{\kappa-1}}{1-\lambda} \right)^{\frac{\kappa}{1-\kappa}}.$$